The Birnie Group solar class and website were created with much-appreciated support from the NSF CRCD Program under grants 0203504 and 0509886. Continuing Support from the McLaren Endowment is also greatly appreciated!

Characteristics of DC Motors

Much appreciation to David Cowden at Ga. Tech.

Archived here from Web citation from 10/15/2004

Slides on these *other topics* might also be of interest (most collected during teaching years 2004 and 2005):

http://www.rci.rutgers.edu/~dbirnie/solarclass/BandGapandDopingLecture.pdf	Band Gap Engineering of Semiconductor Properties
http://www.rci.rutgers.edu/~dbirnie/solarclass/MultijunctionLecture.pdf	Multi-Junction Solar Device Design
http://www.rci.rutgers.edu/~dbirnie/solarclass/MBEgrowth.pdf	Molecular Beam Epitaxy
http://www.rci.rutgers.edu/~dbirnie/solarclass/amorphousSi.pdf	Amorphous Silicon Solar Cells
http://www.rci.rutgers.edu/~dbirnie/solarclass/TransparentConductors.pdf	Transparent Conductors for Solar
http://www.rci.rutgers.edu/~dbirnie/solarclass/ARCoatings.pdf	Anti-Reflection Coatings for Solar
http://www.rci.rutgers.edu/~dbirnie/solarclass/OrganicPV.pdf	Organic PV
http://www.rci.rutgers.edu/~dbirnie/solarclass/DSSC.pdf	Dye Sensitized Solar Cells
http://www.rci.rutgers.edu/~dbirnie/solarclass/2005ProjectResultsindex.htm	Examples of Previous Years' Student Solar Projects

Note: in some cases it may be possible to design custom courses that expand on the above materials (send me email!)

Journal Publications of Some Recent Research: (best viewed through department home index: http://mse.rutgers.edu/dunbar_p_birnie_iii)

Other Birnie Group Research:

Sol-Gel Coating Quality and Defects Analysis (mostly Spin Coating):	http://www.coatings.rutgers.edu
Solar Research at Rutgers: Broader Overview	http://www.solar.rutgers.edu
Solar and Electric Vehicles System Projects (early stage emphasis)	http://www.rave.rutgers.edu

RUTGERS UNIVERSITY

Professor Dunbar P. Birnie, III (dunbar.birnie@rutgers.edu)
Department of Materials Science and Engineering
http://mse.rutgers.edu/faculty/dunbar_p_birnie

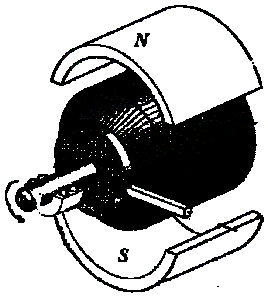
Much appreciation to David Cowden at Ga. Tech.: Web citation from 10/15/2004

http://www.srl.gatech.edu/education/ME3110/primer/motors.htm

Motors

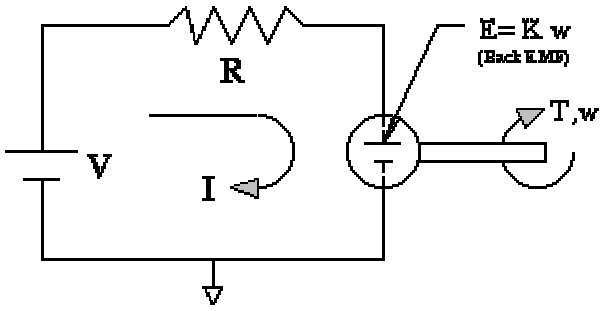
Introduction

This section contains information about implementing dc motors in a mechanical system. Pimary emphasis is placed on DC permanent magnet motors, since these motors are the most common, least expensive, and lightest type of motors.



Elementary Theory of DC Permanent Magnet Motors

Since this type of motor uses a permanent magnet to generate the magnetic field in which the armature rotates, the motor can be modeled by the electrical circuit in the armature alone. If we further simplify the circuit by ignoring the inductance of the armature coil, we arrive with the following circuit diagram for a simple motor:



V is the voltage supplied by the power source (usually a battery), and R is the resistance of the motor's armature coil. This resistance cannot be measured at rest, since it changes as speed increases towards the steady state speed. Kirchoff's voltage law leads to the following equation:

$$V = IR + E$$

By examining the effect of the magnetic field in the motor, and realizing that magnetic flux is constant, we can arrive at the following two equations relating the torque and speed output of the motor to the supplied current and voltage:

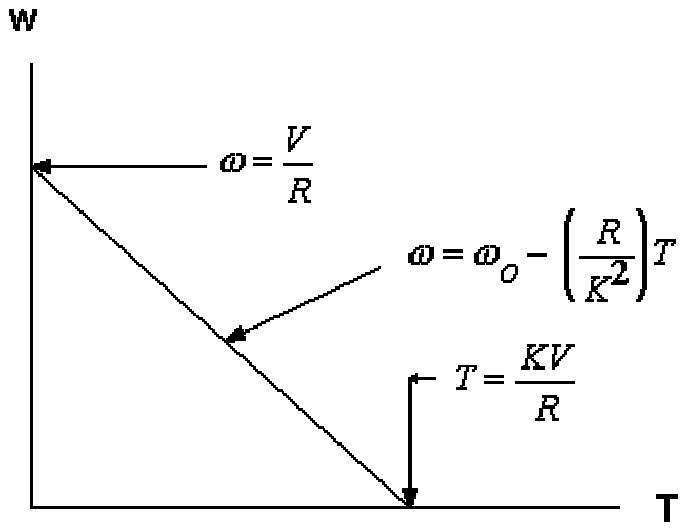
$$E = K_v \cdot \omega \qquad T = K_m \cdot I$$

These are often known as the *transducer equations* for a motor, since a motor is really an electro-mechanical transducer. The constants Kv and Km are dependent on the particular motor, but if they are expressed in **SI UNITS**, their values are always equal, thus

$$\omega = \left(\frac{V}{K}\right) - \left(\frac{R}{K^2}\right) T$$

(note this equation corrected from original web citation)

It is important to remember that, in this equation, T is the total torque acting on the motor, and is generally non-zero even with no applied load, due to the internal losses in the motor. This equation implies that the speed of the motor is equal to some constant, which is a function of the applied voltage and the motor constant, minus another constant times the applied torque. This is an important relation, since it reveals that for this motor model, **Torque is a linear function of speed.** If torque is plotted vs. speed for a motor, the plot will typically look like this:



(note that y-axis intercept definition is wrong. It should read w_o=V/K)

The intercepts of the line at the T and w axes are especially important, since they give the values of the **stall torque** and the **no-load speed**.The stall torque is the torque which the

motor provides when the rotor of the motor is held at zero velocity. It is not difficult to derive from the diagram holding speed and torque, respectively, at zero, that:

$$T_{stall} = \frac{K \cdot V}{R} \qquad \omega_{noload} = \frac{V}{K}$$

It is **very** important to note that in our development, we have included the Torque on the motor, T, as only one term, which includes both a load torque and the torque that is provided by the bearings of the motor and the frictional losses within the motor. The consequence is that, in practice, the torque on the motor is never really zero, and the no-load speed is really only *approximately* given by the relation above. Similar arguments apply to the stall torque, although in this case the difference is much smaller since there are no frictional losses in a motor when the speed is zero.

Since we commonly find the no-load speed given when we buy a motor, we can substitute the no-load speed into the general equation above and find a bit more useful relation for the loaded speed of a motor:

$$\omega = \omega_o - \left(\frac{R}{K^2} \right) T$$

Finally, by taking the derivative of this equation with respect to power, we can find that the maximum power that can be output by the motor is given by the relation:

$$P_{\max} = \left(\frac{1}{4} \right) \left(\frac{V^2}{R} \right)$$

This equation is especially useful when evaluating how large a motor might be required for an application.

Limitations of this motor model

There are several assumptions that we have made in the development of this model, which are very important to keep in mind when using the above relations to implement a dc permanent magnet motor:

1. The torque on the motor has all been lumped into one term, which **includes the torque provided by the bearings of the motor and other frictional losses**. The consequence of this assumption means that in general, *the no-load speed will only*

approximately be given by the relation w=V/K, since in practice a small, non-zero frictional load is always present.
2. We have assumed that the inductance of the motor is zero, which is normally a good assumption (especially for steady state), but must be re-evaluated if transient response of the motor is to be analyzed.
3. We have assumed that the resistance of the motor is constant, which is ideally not true. The resistance of the motor actually changes slightly with speed, and also with temperature. These changes are normally small, however, so this is a valid assumption unless the motor is operating at very high temperatures
4. We have assumed in the development that we actually know the motor constant, K, and the resistance, R, which is in general NOT the case. The motor specifications section is devoted to characterizing motors given information which we in practice are able to obtain.

Motor Specifications

Normally, when you obtain a motor from a commercial source, you will not immediately know all the information necessary to characterize the motor in terms of the theoretical development above. Some motors include the Stall Torque, the no-load speed, and the Resistance, while others include only the rated voltage and the power output at maximum power. In these cases, there are several things you can do to experimentally determine the motor constant, internal resistance of the motor, etc. Here are some basic steps to take to characterize your motor:

1. *Look on the package of the motor, and obtain all the information you can from there. Then use the given information in the above equations to calculate the values of K, R, or other values that you don't know. For example:*

    - Usually at least the Voltage and the no-load speed are given. If they are, you can use the above equations to calculate K=w*V. Be sure to use the correct units.

    - If the stall torque is given, and you have already found K from step 1, you can find the value of R, the internal resistance of the motor.

    If the package of your motor does not contain the information you need, you can always contact the manufacturer, who will be able to give you all the details of the motor that you require.

2. *If step one did not yield enough information to calculate all the parameters, you can test the motor to find some of the values yourself:*

- Use an ammeter (measures current) to measure the current the motor draws. Apply a voltage to the motor, and allow the motor to run with no load. Measure the *no load current*. From above, knowing K, the motor constant, we can calculate the friction torque, Tf, on the motor is I(no load)*K. This torque is present at all times when the motor is running. (It is actually not constant with speed, but it is small, and we can assume without much loss in accuracy that this is the case).

- If you have access to a tachometer, measure the no-load speed, and from the measured Voltage applied calculate K. You can also measure the no-load speed if you have a strobe light by adjusting the strobe period until the motor appears to "stand still".

- Attach a large gear to your motor, and hang a weight from a string that will be wrapped up as the motor turns. At the bottom of the string, hang an object of known mass, or a cup that can hold water. Then, apply a voltage, and continue to increase the voltage until the motor just supports the weight of the object. By calculating the weight of the object and the radius of the gear, you can calculate the *stall torque* of the motor. You know the Voltage applied, so from the relations above you can then calculate (since w=0) that R=Tstall/(K*V). This type of test is called the *locked rotor test*. Another way to perform this test is by applying a constant voltage and varying the applied weight until the stall torque is reached.

3. *Finally, if you cannot obtain the information from the motor itself, and for some reason you cannot obtain the information using the above tests, you can get an idea of the performance of your motor by using information from other motors. For example, if another motor where you got yours has the internal resistance, you could guess that the resistance of your motor is similar, and make the rest of your analysis more conservative to account for the error.*

References

1. Shigley, J.E, and Mischke, C.R., Mechanical Engineering Design, 5th Ed., McGraw-Hill, New York 1989.
2. Buchsbaum, Frank, Design and Application of Small Standardized components Data Book 757 Vol. 2, Stock Drive Products, 1983 (Free copies available at Stock Drive Products).