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Homework #1 – Solutions comments – to match with "Note" numbers in EXCEL file.

Working from the assignment we are solving for Ef in this equation – with all other things known:

$$N_C \exp\left(-\frac{E_C - E_F}{kT}\right) = \frac{N_D}{1 + 2\exp\left(\frac{E_F - E_D}{kT}\right)} + N_V \exp\left(\frac{E_V - E_F}{kT}\right)$$

What we do in EXCEL is rearrange this equation to make a function where we want to solve for where that function becomes ZERO. Here is that function as rearranged:

$$Function(E_F) = N_C \exp\left(-\frac{E_C - E_F}{kT}\right) - N_V \exp\left(\frac{E_V - E_F}{kT}\right) - \frac{N_D}{1 + 2\exp\left(\frac{E_F - E_D}{kT}\right)}$$

I have put the terms in the same order that I have used them in solving the EXCEL spreadsheet.

Comment: The EXCEL spreadsheet was useful for doing this calculation – but recognize that the way that I present below is NOT the only way to do it. You should have been able to get the answer with a variety of different methods.

Here are the notes from the EXCEL:

Note 1: I have defined the constants to be specific cell values at the top of the spreadsheet. So if I wanted to try other values of band gap or something then it would only require changing one cell! I have used the "Insert - Name - Define" to give each of the A cells named constant status and I've put those names in the adjacent cells (column B)

Note 2: Because of the symmetry of the problem (a pn-junction with donor and acceptor concentration being equal) then I carry through with a calculation of only the donor side. Then I will double the answer to arrive at the full $V_{bi}$ values.

Note 3: instead of really working through the derivative that you need to calculate, we will do it numerically. The Fermi energy will be solved for two temperatures and then the fractional change will be found by dividing by the temperature difference – I'll clarify this more below, but these cells give the T and dT that are used in the following

calculation. Again, these are individual cells that govern the following calculations – so it would only be needed to change one cell to reach the new answer. Test it out.

Note 4: Column C has the function calculation at T=300 and Column D has the function calculation at T=301. Column A has the doping level in each cell – the **BOLD** cell is one where you can plug in values and see how it changes things. Column B is important – this is where I *test* different possible $E_F$ values and then find out if the function is close to zero. When we find an $E_F$ value that forces the function to be zero then we have found the answer for the true Fermi energy. Since we don't really know what the $E_F$ value will be then I have written this spreadsheet to test ALL of them at 0.001 eV intervals. This means that I've copied the function 550 times and gradually vary the $E_F$ along the way.

> (This process can be achieved by using GoalSeek – and this is the way that I coached you all to do it, but because of difficulty finding zeros and possible uncertainty in interpreting those numbers then I am presenting a more brute-force approach.)

Note 5: Since we don't know what the $E_F$ answer will be, I use a trick to find what the answer will be. It is based on the knowledge that the function being evaluated will be negative on one side of the answer and positive on the other side of the answer. So I have inserted an "IF" statement in Column E that helps find where the zero crossing is. The function in cell E18 is this: **=IF(C18<0,1,0)** -- this returns a value of 1 if the evaluation function is less than zero and returns a 0 if the function is greater than zero. Cell G18 then directs us to the energy value right after the zero crossing. Cell G18 is simply the sum of all of the test results in column E. So the value in G18 is the number of rows that have negative function value – and since we have incremented the energy by 0.001, then the quantity is an estimate of the fermi energy in millivolts. This technique is very useful in processing things in EXCEL spreadsheets.

Note 6: Since we have found where the zero crossing is – I want to be more precise about exactly what the Fermi energy is – better than the 0.001 volt resolution. So I have copied a segment of the set of columns to the left and we will do the same calculation again, but ONLY around the energy of interest. Thus the functions are made to reference Nd and Ef values in columns H and I – and the top cell in H is referenced to the column A value. Then the $E_F$ is referenced to the value in cell G18 – so the value of cell I25 is then chosen to be the value that is 7 mV less and with 14 rows altogether then we will easily straddle the true exact zero crossing. Note that this set of 14 rows works around the zero crossing AUTOMATICALLY. So that when we change the doping level (the thing that we had to do as the main part of the assignment) then this will arrive at the answer rapidly.

Note 7: Since we are looking so closely around the spot where the zero crossing happened then we can use a linear regression to find the best $E_F$ answer. EXCEL has both Slope and intercept functions. If we choose the E's as the Y values and the function as the X values then we only need to use the intercept function. So cell J44 has this function: "**=INTERCEPT(I25:I38,J25:J38)**". Then the cell K44 has the same intercept calculated for the T=301 function. Note the very slight difference in fermi energy for the two lines.

Note 8: This is a plot of the key segment of the spreadsheet around the zero crossing. Note the slight shift to the left as the temperature is increased – the Fermi energy moves closer to zero – closer to mid-gap – as the temperature goes up.

Note 9: (shown below the graph – you will need to scroll down). The two built-in-voltages are calculated (as simply 2X the fermi levels found in Note 8). This is because the p-type fermi level is exactly as far BELOW the mid gap point as the n-type Fermi level is above the mid gap point.

Finally, the fractional change in power is

$$\frac{1}{Power} * \frac{d(Power)}{dT} = \frac{1}{V_{bi}} * \frac{d(V_{bi})}{dT} = \frac{1}{V_{bi}} * \frac{\Delta(V_{bi})}{\Delta T}$$

So cell H73 does this calculation (at least the ratios at the right side of the equation).

Note 10: (this is back up near the top in cell J12.) The plot is supposed to be the fractional power change as a function of the Log10(doping level). Thus I have placed two side-by-side cells that have these values (J15 and K15) where J15 evaluates the master value from cell A18 and K15 just copies the value from H73 below. Having them sidebyside then makes it easier to copy the values into a different spreadsheet page to allow for plotting.

So the sheet named "AnswerSheet" has the numerical values evaluated at several points along the range that was assigned. The sheet named "AnswerGraph" has the plot that I asked you to make – this is simply a plot of the values in AnswerSheet.

> Interpretation: The general tendancy is that more heavily doped material has smaller voltage change with temperature. All of the materials have a power **reduction** with increasing temperature. The amount of power reduction is rather modest – requiring 10's of degrees to really cause major power loss, though this level of temperature rise really could occur if measures are not take to conduct away the waste heat that comes from the incident sunlight (the part that doesn't get converted into electrical power).